Spectrum Allocation and Subsidization for User Welfare in Mobile Communication Services

Seung Min Yu and Seong-Lyun Kim

***Abstract*—Mobile traffic explosion causes spectrum shortage and polarization of data usage among users, which will eventually decrease user welfare in mobile communication services. Therefore, governments around the world are planning to make more spectrum available for mobile broadband use, and the key policy issue is to find an efficient spectrum allocation method that will improve user welfare. In this paper, we propose a data subsidy scheme where the regulator offers a spectrum price discount to mobile network operators (MNOs) in return for imposing the responsibility of providing a predefined data amount to users free of charge. To mathematically analyze the subsidy effect, we take the two-stage Cournot and Bertrand competition model and apply the equilibrium analysis based on the game-theoretic approach. Consequently, we show that the data subsidy scheme increases user welfare even further. An interesting observation is that the increase in user welfare does not involve MNO profit loss and the increasing amount is higher than the regulator's expenses for implementing the data subsidy scheme.**

***Index Terms*—Network economics, game theory, spectrum policy, subsidy, competition, mobile communications.**

I. Introduction

Mobile data traffic grows exponentially with emergence of smart devices (e.g., smart phones and tablet PCs). According to a Cisco report, global mobile data traffic will increase 26-fold between 2010 and 2015 [1]. This growth drives mobile network operators (MNOs) to purchase more spectrum amount because it is a straightforward way to enhance network capacity. However, the amount of usable spectrum is limited and MNOs compete with each other. This provokes increasing spectrum price, which will eventually lead to a high service price for users. Recently we investigated price competition between network service providers and suggested regulation rules for achieving Pareto optimal equilibria [2], [3].

Data traffic growth can cause polarization of data usage among users. Data traffic usage behavior follows the Pareto principle, where 20 percent of heavy users consume 80 percent of the total traffic [4]. Some people cannot use network services because they are economically unstable, or live in rural areas. For this reason, the European Commission recommended that the Universal Service Obligation (USO) be placed on MNOs to ensure that services are accessible to all people within a reasonable price range [5]. Since Switzerland firstly imposed the USO in 2008, many countries have adopted it. Even though the USO reduces the regional imbalance in traffic usage, the imbalance resulting from income inequality still remains.

The other aspect is how the regulator (government) improves user welfare within mobile communication services. A straightforward way is to allocate additional spectrum. Regulators around the world are planning to make more spectrum available for mobile broadband use. Particularly, the U.S. Federal Communications Commission (FCC) recently released the National Broadband Plan, which calls for 300 MHz additional spectrum in the short-term and 500 MHz in the long-term [6]. In these circumstances, a key policy issue is to find an efficient spectrum allocation method that will improve user welfare. One argument suggests providing the additional spectrum in unlicensed form. However, a previous work [7] pointed out that additional unlicensed spectrum allocation might be inefficient, owing to the high congestion level caused by many free users (i.e., *the tragedy of commons*).

In this paper, we propose a *data subsidy scheme* for improving user welfare while guaranteeing profits to MNOs. In this scheme, the regulator offers a *spectrum price discount* to MNOs in return for imposing the responsibility of providing a predefined data amount to users free of charge. Limited free data service plays important roles in both increasing user welfare and avoiding the tragedy of commons by driving users to consume data rationally. We will mathematically show that the data subsidy scheme increases user welfare without MNO profit loss and the increasing amount is higher than the regulator's expenses for implementing the scheme.

II. Subsidization in Mobile Communication Services

A simple subsidization method is to give users money as a subsidy. We call this the *price subsidy scheme*, where the regulator gives an equal price subsidy $\hat{p}$ to all users that consume network services. Then, every user's willingness-to-pay increases and the total traffic demand changes. Figure 1 illustrates change in total traffic demand with the price subsidy scheme. The original demand curve is translated by $\hat{p}$, parallel to the price axis. MNOs can absorb all the price subsidy amount by simply increasing their service prices by $\hat{p}$. The price subsidy scheme may not necessarily lead to an increase in the sum of all users' utilities (i.e., user welfare).

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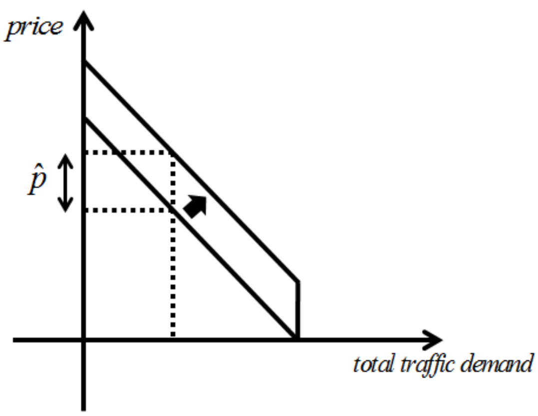

Fig. 1. Total traffic demand for network services in the price subsidy scheme.

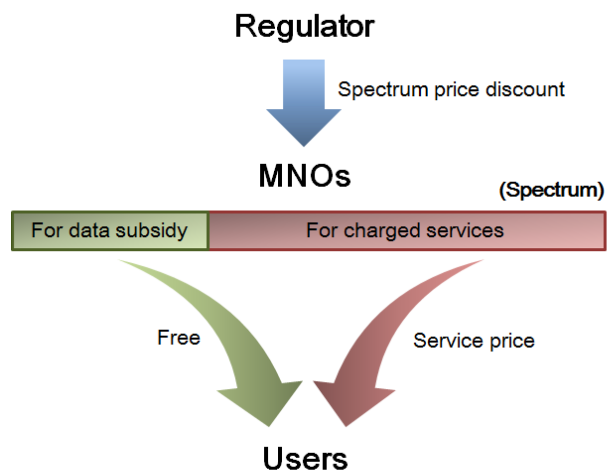

Fig. 2. Spectrum allocation and price in the data subsidy scheme.

The data subsidy scheme we propose for improving user welfare is illustrated in Figure 2. With this, MNOs should provide a predefined data amount to users free of charge. The free data amount is decided by the regulator. The data subsidy reduces the total traffic demand for charged network services, which will eventually decrease MNO revenue. To compensate for this, the regulator offers a spectrum price discount to MNOs. From the regulator perspective, an important problem is how to determine the subsidy amount and the discount rate for the spectrum. This will be mathematically analyzed and discussed in Section V.

To analyze the subsidy effect under the competitive environment reflecting the reality, we take a two-stage competition model considering the combination of Cournot and Bertrand competition models [8]. In the Cournot model [9], MNOs compete with each other by purchasing spectrum to improve their network capacities. On the other hand, in the Bertrand model [10], MNOs engage in price competition to attract more users for a given network capacity. We use the Cournot and Bertrand models so that the spectrum amount is determined in the Cournot stage and afterwards the service price is determined in the Bertrand stage.

As preliminary steps to Section V, we describe the two-stage competition model and characteristics of user demand in Section III. MNO profit maximization in the two-stage duopoly competition is explained in Section IV. Finally in Section V, we quantify the gain of the data subsidy scheme based on the results of Sections III and IV. We extend the duopoly competition into the oligopoly competition in Section VI.

III. Two-Stage Competition and User Demand

A. Two-Stage Competition

Consider a service area covered by two MNOs. Let $i \in \{1, 2\}$ and $-i \in \{1, 2\}$ denote the decision maker and its competitor indexes, respectively. The MNOs compete with each other to attract more users and to maximize their profits. To analyze this, we use the two-stage competition model from Figure 3.

In the Cournot stage, each MNO purchases the spectrum to increase its network capacity. Let w_i and k_i denote MNO i's spectrum purchasing amount and network capacity, respectively. The network capacity is determined by not only the spectrum amount but also the spectral efficiency in the service area. We assume that both MNOs use the orthogonal frequency division multiple access (OFDMA) technology to avoid mutual interferences among their users, and their technological prowess is similar. Then, MNO i's network capacity is

$$k_i = \alpha w_i, \tag{1}$$

where α denotes the spectral efficiency (bps/Hz). Higher network capacity enables MNOs to provide more network services but it costs more. We denote the unit spectrum cost by c. Then, MNO i's total spectrum cost is equal to cw_i.

In the Bertrand stage, MNOs compete with each other by setting their service prices under the given capacities (determined in the Cournot stage). Let p_i and p_{-i} denote MNO i's service price and its competitor's, respectively. The demand for MNO i is a function of p_i and p_{-i} denoted by $d_i(p_i, p_{-i})$. Then, MNO i's revenue is equal to $d_i(p_i, p_{-i})\, p_i$.

B. User Demand

User demand for a MNO satisfies the following assumptions: First, the user demand is a decreasing function of the service price. We assume that, for mathematical tractability, $d_i(p_i, p_{-i})$ is a linearly decreasing function of p_i. Second, people prefer a low price and user demand for MNO i is affected by its competitor's service price. If p_i is lower than p_{-i}, then $d_i(p_i, p_{-i})$ is independent of p_{-i}. On the other hand, if p_i is higher than p_{-i}, then $d_i(p_i, p_{-i})$ is the residual demand that cannot be served by MNO $-i$ owing to capacity limitation. In the equal price case, we assume that the demand for MNO i is proportional to the network

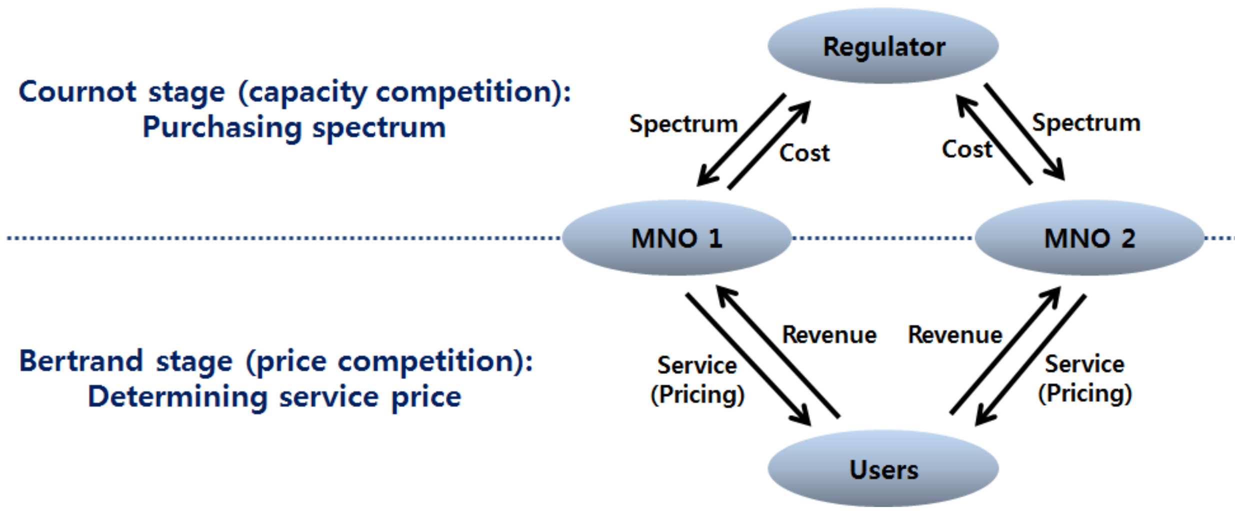

Fig. 3. Two-stage Cournot and Bertrand competition between two MNOs.

capacity [11]. Then, $d_i(p_i, p_{-i})$ is expressed as follows:

$$d_i(p_i, p_{-i}) = \begin{cases} \bar{d}\left(1 - \frac{p_i}{\bar{p}}\right), & \text{if } p_i < p_{-i}, \\ \bar{d}\left(1 - \frac{p_i}{\bar{p}}\right) - \alpha w_{-i}, & \text{if } p_i > p_{-i}, \\ \frac{w_i}{w_i + w_{-i}}\bar{d}\left(1 - \frac{p_i}{\bar{p}}\right), & \text{if } p_i = p_{-i}, \end{cases} \quad (2)$$

where the given parameters $\bar{d}$ and $\bar{p}$ denote the maximum demand and the maximum willingness-to-pay, respectively. Figure 4 illustrates the demand function $d_i(p_i, p_{-i})$.

IV. Duopoly Competition

In this section, we analyze the "*conventional*" competition between two operators, based on the two-stage model and game theoretic approaches. Our analysis here is extended to the data subsidy case in Section V, to derive the gain of the subsidy scheme compared to the conventional competition. For this purpose, let us start with the Bertrand stage, where the capacity k_i is a given value.

A. Bertrand Stage: Price Competition

Let us formulate MNO i's profit maximization problem in the Bertrand stage as follows:

$$\begin{aligned} \max_{p_i} \quad & d_i(p_i, p_{-i}) p_i - c w_i, \\ \text{s.t.} \quad & d_i(p_i, p_{-i}) \le \alpha w_i. \end{aligned}$$

Note that w_i is a given value, and the network capacity (αw_i) and the total cost ($c w_i$) are fixed. Thus, the MNO has to maximize its revenue ($d_i(p_i, p_{-i}) p_i$) by controlling p_i under the capacity constraint. We derive MNO i's optimal price, as summarized in the following lemmas:

Lemma 1: *In the case that MNO i's price (p_i) is lower than its competitor's price (p_{-i}), the optimal price (p_i^*) is*

$$p_i^* = \begin{cases} \bar{p}\left(1 - \frac{\alpha w_i}{\bar{d}}\right), & \text{if } p_{-i} > \bar{p}\left(1 - \frac{\alpha w_i}{\bar{d}}\right), \\ p_{-i} - \varepsilon, & \text{if } p_{-i} \le \bar{p}\left(1 - \frac{\alpha w_i}{\bar{d}}\right), \end{cases}$$

where ε is a small positive value.

Proof: From Equation (2), we calculate MNO i's revenue $d_i(p_i, p_{-i}) p_i$ in this case ($p_i < p_{-i}$):

$$d_i(p_i, p_{-i}) p_i = \bar{d}\left(p_i - \frac{p_i^2}{\bar{p}}\right). \quad (3)$$

However, the MNO cannot satisfy the higher demand than its network capacity. Reflecting this, we rewrite the revenue as follows:

$$d_i(p_i, p_{-i}) p_i = \min\left\{\alpha w_i p_i, \bar{d}\left(p_i - \frac{p_i^2}{\bar{p}}\right)\right\}. \quad (4)$$

In the minimum operator of the revenue, the left side is a linear function whose slope is αw_i (MNO i's network capacity)[1] and the right side is a quadratic function whose maximum is at $p_i = \bar{p}/2$. The intersection of these functions is at $p_i = \bar{p}(1 - \alpha w_i/\bar{d})$. Therefore, if the competitor's price p_{-i} is higher than $\bar{p}(1 - \alpha w_i/\bar{d})$, the optimal solution is $p_i^* = \bar{p}(1 - \alpha w_i/\bar{d})$. Otherwise, the optimal solution is $p_i^* = p_{-i} - \varepsilon$, where ε denotes a minimum unit of price level changes. ■

Lemma 2: *In the case that MNO i's price (p_i) is higher than its competitor's price (p_{-i}), the optimal price (p_i^*) is*

$$p_i^* = \begin{cases} \bar{p}\left(1 - \frac{\alpha(w_i + w_{-i})}{\bar{d}}\right), & \text{if } p_{-i} < \bar{p}\left(1 - \frac{\alpha(w_i + w_{-i})}{\bar{d}}\right), \\ p_{-i} + \varepsilon, & \text{if } p_{-i} \ge \bar{p}\left(1 - \frac{\alpha(w_i + w_{-i})}{\bar{d}}\right). \end{cases}$$

Proof: Similar to the proof on Lemma 1, we calculate MNO i's revenue in this case ($p_i > p_{-i}$) as follow:

$$d_i(p_i, p_{-i}) p_i = \min\left\{\alpha w_i p_i, \bar{d}\left(p_i - \frac{p_i^2}{\bar{p}}\right) - \alpha w_{-i} p_i\right\}.$$

In the minimum operator of the revenue, the left side is a linear function whose slope is αw_i and the right side is a quadratic function whose maximum is at $p_i = \bar{p}/2(1 -$

[1] Here we assume that MNO i's network capacity αw_i (determined in the Cournot stage) is much less than the maximum demand $\bar{d}$, which affects MNO i's optimal solution in the Bertrand stage. This assumption is justified by the mobile traffic explosion and the spectrum shortage where the spectrum cost c is very high.

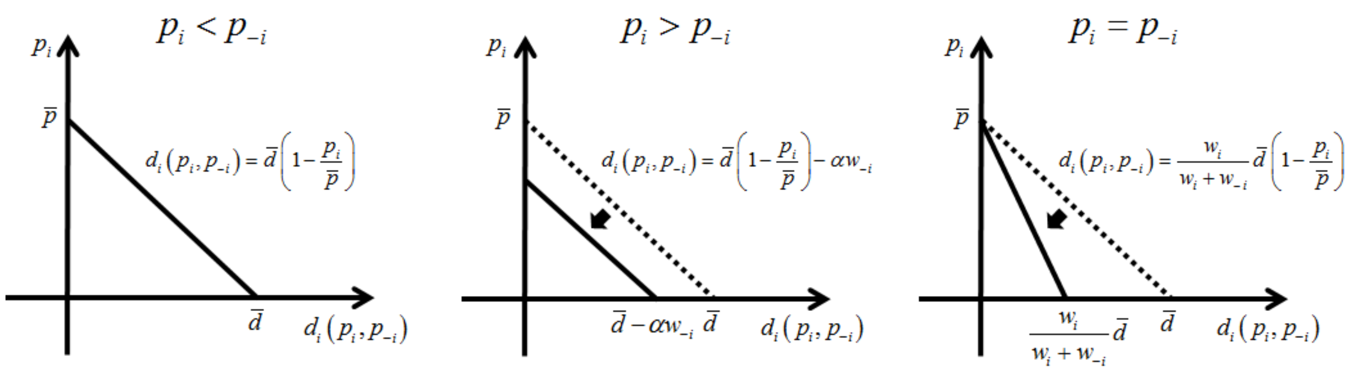

Fig. 4. Demand function for MNO i's service under duopoly competition.

$\alpha w_{-i}/\bar{d})$. The intersection of these functions is at $p_i = \bar{p}(1-\alpha(w_i+w_{-i})/\bar{d})$. Therefore, if the competitor's price p_{-i} is lower than $p_i = \bar{p}(1-\alpha(w_i+w_{-i})/\bar{d})$, the optimal solution is $p_i = \bar{p}(1-\alpha(w_i+w_{-i})/\bar{d})$. Otherwise, the optimal solution is $p_i^* = p_{-i} + \varepsilon$. ■

MNO i's optimal price p_i^* is a function of the competitor's price p_{-i}, and there will be a strategic interaction between the two MNOs. Therefore, we use the Nash equilibrium (NE) concept to predict the result of the strategic interaction [12], [13]. NE represents an equilibrium point of non-cooperative game, where no one has anything to gain by changing their own strategy. Using Lemmas 1 and 2, we find a NE in the Bertrand stage as follows:

Proposition 1: *In the Bertrand stage where both MNOs' spectrum amounts (w_1 and w_2) are given, there is a Nash equilibrium (p^{NE}) in prices:*

$$\begin{aligned} p^{NE} &= \left(p_1^{NE}, p_2^{NE}\right) \\ &= \left(\bar{p}\left(1 - \tfrac{\alpha(w_1+w_2)}{\bar{d}}\right), \bar{p}\left(1 - \tfrac{\alpha(w_1+w_2)}{\bar{d}}\right)\right). \end{aligned}$$

Proof: If $p_i = p_{-i} = \bar{p}(1-\alpha(w_i+w_{-i})/\bar{d})$, the demand for MNO i is:

$$\begin{aligned} d_i\left(p_i, p_{-i}\right) &= \frac{w_i}{w_i + w_{-i}}\bar{d}\left(1 - \frac{p_i}{\bar{p}}\right) \\ &= \frac{w_i}{w_i + w_{-i}}\bar{d}\left(\frac{\alpha\left(w_i + w_{-i}\right)}{\bar{d}}\right) \\ &= \alpha w_i. \end{aligned} \tag{5}$$

In Equation (5), the demand is equal to the network capacity. In this case, even if MNO i lowers its price, its revenue will not increase because MNO i cannot serve more traffic owing to the capacity limitation. If MNO i applies a strategy to raise its price, then the optimal choice is to increase the price by ε, but this leads to the revenue decrease (Lemma 2). Therefore, MNO i has no motivation to deviate from the price $p_i = p_{-i} = \bar{p}(1-\alpha(w_i+w_{-i})/\bar{d})$. ■

B. Cournot Stage: Capacity (Spectrum) Competition

In the Cournot stage, MNO i's profit maximization problem is:

$$\begin{aligned} \max_{w_i} \quad & d_i\left(p_i\left(w_i, w_{-i}\right), p_{-i}\left(w_i, w_{-i}\right)\right) p_i\left(w_i, w_{-i}\right) - c w_i, \\ \text{s.t.} \quad & d_i\left(p_i\left(w_i, w_{-i}\right), p_{-i}\left(w_i, w_{-i}\right)\right) \le \alpha w_i, \end{aligned}$$

where MNOs determine the spectrum amount to purchase, which affects the NE in the Bertrand stage. With Proposition 1, we rewrite the profit maximization problem as follows:

$$\max_{w_i} \quad \alpha w_i \bar{p}\left(1 - \frac{\alpha\left(w_i + w_{-i}\right)}{\bar{d}}\right) - c w_i,$$

of which the optimal solution is summarized in the following lemma:

Lemma 3: *If the competitor's spectrum amount w_{-i} is given, MNO i's optimal spectrum amount (w_i^*) is*

$$w_i^* = \frac{\bar{d}}{2\alpha}\left(1 - \frac{c}{\alpha\bar{p}}\right) - \frac{1}{2}w_{-i}.$$

Proof: The objective function is a quadratic function and its first-order derivative is a linear function of w_i. Therefore, we can find the optimal solution by solving a linear equation. ■

With Lemma 3, we find a NE in the Cournot stage as follows:

Proposition 2: *In the Cournot stage, there is a Nash equilibrium (w^{NE}) in spectrum amounts:*

$$w^{NE} = \left(w_1^{NE}, w_2^{NE}\right) = \left(\tfrac{\bar{d}}{3\alpha}\left(1 - \tfrac{c}{\alpha\bar{p}}\right), \tfrac{\bar{d}}{3\alpha}\left(1 - \tfrac{c}{\alpha\bar{p}}\right)\right).$$

Proof: Using the fact that an intersection of all players' optimal strategies becomes a NE, the following equations should be satisfied:

$$\begin{aligned} w_1^{NE} &= \frac{\bar{d}}{2\alpha}\left(1 - \frac{c}{\alpha\bar{p}}\right) - \frac{1}{2}w_2^{NE}, \\ w_2^{NE} &= \frac{\bar{d}}{2\alpha}\left(1 - \frac{c}{\alpha\bar{p}}\right) - \frac{1}{2}w_1^{NE}. \end{aligned}$$

Therefore, we can find a NE by solving the linear simultaneous equations. ■

We have found a NE in each of the Cournot and Bertrand stages separately. However, these two stages are interlinked and the equilibriums should be integrated. In game theory, subgame perfect equilibrium (SPE) is a refined concept of NE used in multi-stage games, which represents a NE of every stage (or subgame) of the original game [12], [13]. No player has anything to gain by changing its own strategy in the SPE, and the strategies in the SPE will probably be the outcome of the strategic interaction between players. Therefore, to describe characteristics of the two-stage competition market, we will find a SPE. For this, we use the backward induction method which aims to solve a multi-stage game backwards from the last stage of the game. With Propositions 1 and 2, we can find a SPE in the two-stage competition as follows:

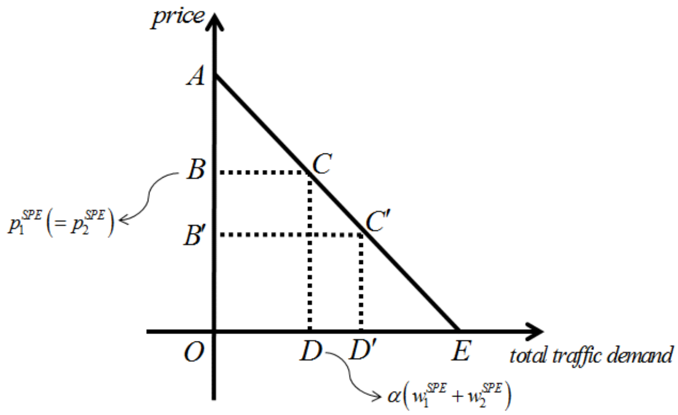

Fig. 5. Total traffic demand for network services in duopoly competition.

Proposition 3: *In the two-stage duopoly competition, there is a subgame perfect equilibrium (*w^{SPE}*,*p^{SPE}*):*

$$w^{SPE}=\left(w_1^{SPE},w_2^{SPE}\right)=\left(\tfrac{\bar{d}}{3\alpha}\left(1-\tfrac{c}{\alpha\bar{p}}\right),\tfrac{\bar{d}}{3\alpha}\left(1-\tfrac{c}{\alpha\bar{p}}\right)\right),$$
$$p^{SPE}=\left(p_1^{SPE},p_2^{SPE}\right)=\left(\tfrac{\bar{p}}{3}\left(1+\tfrac{2c}{\alpha\bar{p}}\right),\tfrac{\bar{p}}{3}\left(1+\tfrac{2c}{\alpha\bar{p}}\right)\right).$$

Proof: We first check that the SPE represents a NE in the last stage (i.e., the Bertrand stage) of the two-stage competition game:

$$\begin{aligned}p_i^{SPE}=&\frac{\bar{p}}{3}\left(1+\frac{2c}{\alpha\bar{p}}\right)=\bar{p}\left(1-\frac{2}{3}+\frac{2c}{3\alpha\bar{p}}\right)\\=&\bar{p}\left(1-\frac{\alpha}{\bar{d}}\left(\frac{2\bar{d}}{3\alpha}\left(1-\frac{c}{\alpha\bar{p}}\right)\right)\right)\\=&\bar{p}\left(1-\frac{\alpha\left(w_i^{SPE}+w_{-i}^{SPE}\right)}{\bar{d}}\right).\end{aligned}\tag{6}$$

Equation (6) shows that the SPE represents a NE in the Bertrand stage (Proposition 1). The SPE also represents a NE in the Cournot stage (Proposition 2). ■

Proposition 3 indicates that both MNOs purchase the same spectrum amount and adopt an equal pricing strategy. Using Equation (2), we can show that the demand for each MNO is equal to its own network capacity in the SPE (i.e., *market clearing*). An interesting observation is that technological innovation (i.e., making the spectral efficiency α high) brings the service price decreasing effect. On the other hand, the high spectrum cost causes MNOs to reduce their investment and to raise their service prices. This means that some users cannot afford to consume network services (i.e., polarization of data usage), which will eventually decrease overall user welfare. More details will be discussed in the next section.

C. Welfare Analysis

To describe each player's welfare in the network service market, we define MNO profit (MP), user welfare (UW) and regulator revenue (RR). MP is the sum of both MNOs' profits. UW is the sum of all users' utilities. If a user consumes a MNO's network service, then his net utility is equal to his willingness-to-pay minus the MNO's service price. If a user dose not consume either of MNOs' network services, then his utility is equal to zero. RR is the regulator's spectrum sales revenue. From the regulator's perspective, spectrum policies that solely improve MP are not desirable if they are at the cost of sacrificing UW.

With Proposition 3, MP, UW and RR are given as follows:

$$\begin{aligned}\text{MP}=&\alpha w_1^{SPE}p_1^{SPE}-cw_1^{SPE}+\alpha w_2^{SPE}p_2^{SPE}-cw_2^{SPE}\\=&\frac{2}{9}\bar{p}\bar{d}\left(1-\frac{c}{\alpha\bar{p}}\right)^2,\end{aligned}\tag{7}$$

$$\begin{aligned}\text{UW}=&\frac{1}{2}\alpha w_1^{SPE}\left(\bar{p}-p_1^{SPE}\right)+\frac{1}{2}\alpha w_2^{SPE}\left(\bar{p}-p_2^{SPE}\right)\\=&\frac{2}{9}\bar{p}\bar{d}\left(1-\frac{c}{\alpha\bar{p}}\right)^2,\end{aligned}\tag{8}$$

$$\begin{aligned}\text{RR}=&c\left(w_1^{SPE}+w_2^{SPE}\right)\\=&\frac{2}{3}\frac{c\bar{d}}{\alpha}\left(1-\frac{c}{\alpha\bar{p}}\right).\end{aligned}\tag{9}$$

From Equations (7) and (8), we observe that increase of the unit spectrum cost c decreases MP and UW. This is because MNOs reduce spectrum purchasing amount and raise service prices in return (Proposition 3). For graphical interpretation, we plot the total traffic demand for network services in duopoly competition (Figure 5). In the figure, points B and D denote the service price and the sum of both MNOs' network capacities in the SPE, respectively. Note that the area of triangle ABC is equal to UW. If the service price decreases to B', then UW will increase to the area of triangle $AB'C'$. However, MNOs have no motivation to lower their service prices because it causes MP loss. Making the spectral efficiency high can decrease the service price (Proposition 3). Unfortunately, in the real world, technological innovation occurs slowly. Therefore, we need an alternative strategy to improve UW. For this, we propose the data subsidy scheme and mathematically analyze its effect in the next section.

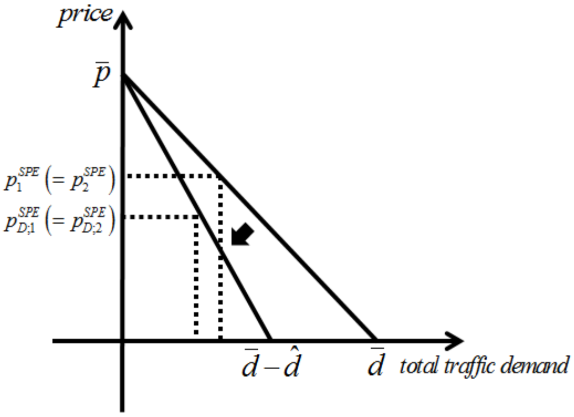

Fig. 6. Total traffic demand for network services in the data subsidy scheme.

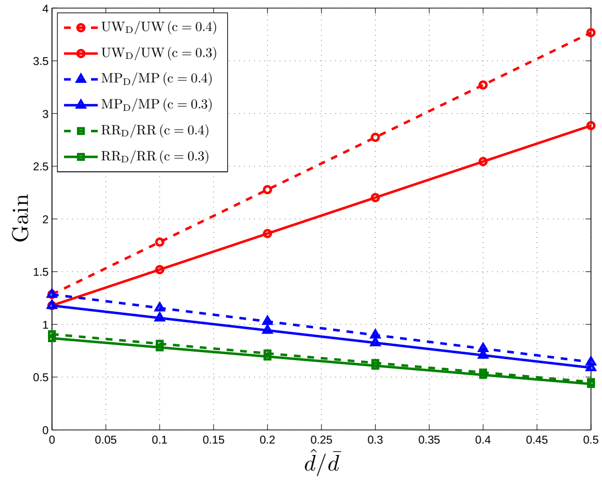

Fig. 7. MNO profit ($\mathrm{MP_D/MP}$), user welfare ($\mathrm{UW_D/UW}$) and regulator revenue ($\mathrm{RR_D/RR}$) gains as a function of the data subsidy level ($\hat{d}/\bar{d}$). Without loss of generality, we assume $\bar{p} = 1$, $\alpha = 1$ and $\delta = 0.2$.

V. Data Subsidy Effect on User Welfare

In the data subsidy scheme, MNOs should provide a predefined data amount $\hat{d}$ to users free of charge. The data subsidy reduces the total traffic demand for charged network services, which will eventually decrease MNO revenues (see Figure 6). On the other hand, the spectrum price discount will decrease MNO costs. We denote the discount rate by δ. From the regulator perspective, an important problem is how to determine $\hat{d}$ and δ for improving UW without any loss in MP. To make an efficient policy, the regulator should also check whether UW gain is larger than the loss in RR.

Following several steps similar to Proposition 3, we can find a SPE in the data subsidy scheme as follows:

Proposition 4: *In the two-stage duopoly competition, if the regulator adopts the data subsidy scheme, then there is a subgame perfect equilibrium (w_D^{SPE},p_D^{SPE}):*

$$
\begin{aligned}
w_D^{SPE} &= \left(w_{D;1}^{SPE}, w_{D;2}^{SPE}\right) \\
&= \left(\tfrac{\bar{d}-\hat{d}}{3\alpha}\left(1-\tfrac{(1-\delta)c}{\alpha\bar{p}}\right), \tfrac{\bar{d}-\hat{d}}{3\alpha}\left(1-\tfrac{(1-\delta)c}{\alpha\bar{p}}\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
p_D^{SPE} &= \left(p_{D;1}^{SPE}, p_{D;2}^{SPE}\right) \\
&= \left(\tfrac{\bar{p}}{3}\left(1+\tfrac{2(1-\delta)c}{\alpha\bar{p}}\right), \tfrac{\bar{p}}{3}\left(1+\tfrac{2(1-\delta)c}{\alpha\bar{p}}\right)\right).
\end{aligned}
$$

Proof: In the data subsidy scheme, the maximum demand for charged network services and the unit spectrum cost become $\bar{d}-\hat{d}$ and $(1-\delta)c$, respectively. Excepting this change, steps for finding a SPE in the data subsidy scheme are equal to that in the conventional scheme (Proposition 3). Therefore, we can find a SPE in the data subsidy scheme by replacing $\bar{d}$ with $\bar{d}-\hat{d}$ and c with $(1-\delta)c$ in Proposition 3. ∎

Proposition 4 shows both MNOs use an equal investing and pricing strategy in the SPE. If the regulator offers the spectrum price discount (δ) to MNOs, then the MNOs purchase more spectrum and lower their service prices to attract more users. This is the reason why the service price becomes a decreasing function of δ in the data subsidy scheme.

With Proposition 4, we calculate MP, UW and RR in the data subsidy scheme as follows:

$$
\begin{aligned}
\mathrm{MP}_D &= \alpha w_{D;1}^{SPE} p_{D;1}^{SPE} - (1-\delta)\, c w_{D;1}^{SPE} + \alpha w_{D;2}^{SPE} p_{D;2}^{SPE} \\
&\quad - (1-\delta)\, c w_{D;2}^{SPE} \\
&= \frac{2}{9}\bar{p}\left(\bar{d}-\hat{d}\right)\left(1-\frac{(1-\delta)\,c}{\alpha\bar{p}}\right)^2, \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{UW}_D &= \frac{1}{2}\alpha w_{D;1}^{SPE}\left(\bar{p}-p_{D;1}^{SPE}\right) + \frac{1}{2}\alpha w_{D;2}^{SPE}\left(\bar{p}-p_{D;2}^{SPE}\right) \\
&\quad + \frac{1}{2}\bar{p}\hat{d} \\
&= \frac{2}{9}\bar{p}\left(\bar{d}-\hat{d}\right)\left(1-\frac{(1-\delta)\,c}{\alpha\bar{p}}\right)^2 + \frac{1}{2}\bar{p}\hat{d}, \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{RR}_D &= (1-\delta)\, c\left(w_{D;1}^{SPE} + w_{D;2}^{SPE}\right) \\
&= \frac{2}{3}\frac{(1-\delta)\, c\left(\bar{d}-\hat{d}\right)}{\alpha}\left(1-\frac{(1-\delta)\,c}{\alpha\bar{p}}\right), \qquad (12)
\end{aligned}
$$

where subscript D denotes the data subsidy scheme. In the data subsidy scheme, the regulator can control MP and UW for his purpose by adjusting the data subsidy amount $\hat{d}$ and the spectrum price discount rate δ. Interestingly, the regulator can increase UW by just giving the spectrum price discount without the data subsidy (i.e., $\hat{d} = 0$), which always involves the increase of MP. On the other hand, if the regulator adopts the data subsidy scheme without the spectrum price discount (i.e., $\delta = 0$), only UW increases. In this case, the regulator should be cautious about determining the data subsidy level because it decreases MP.

For more discussion, we plot MP, UW and RR gains (i.e., $\mathrm{MP_D/MP}$, $\mathrm{UW_D/UW}$ and $\mathrm{RR_D/RR}$, respectively) by adopting the data subsidy scheme in Figure 7. In the figure, other parameters are fixed as $\bar{p} = 1$, $\alpha = 1$ and

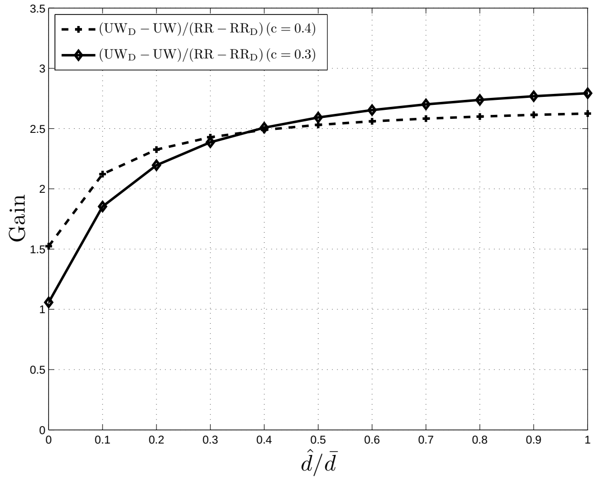

Fig. 8. Ratio of user welfare increase to regulator revenue decrease as a function of the data subsidy level ($\hat{d}/\bar{d}$). Without loss of generality, we assume $\bar{p}=1$, $\alpha=1$ and $\delta=0.2$.

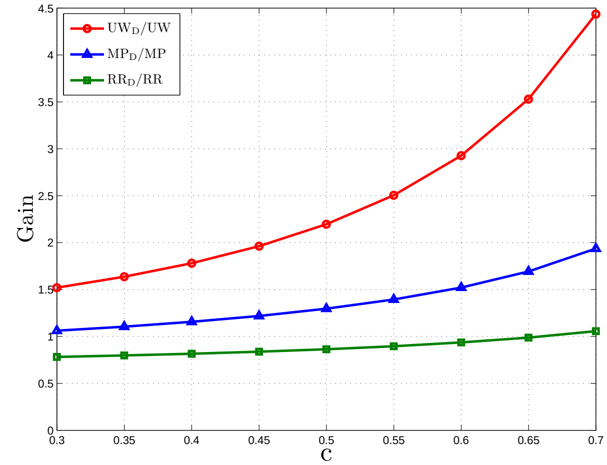

Fig. 9. MNO profit ($\mathrm{MP_D}/\mathrm{MP}$), user welfare ($\mathrm{UW_D}/\mathrm{UW}$) and regulator revenue ($\mathrm{RR_D}/\mathrm{RR}$) gains as a function of the unit spectrum cost c. Without loss of generality, we assume $\bar{p}=1$, $\alpha=1$, $\delta=0.2$ and $\hat{d}/\bar{d}=0.1$.

$\delta=0.2$. The figure shows that UW gain increases with the data subsidy level. On the other hand, MP and RR gains decrease as the data subsidy level increases. RR gain is always lower than 1 because RR decrease is unavoidable in providing the data subsidy and the spectrum price discount. An interesting observation is that MNOs can profit more or avoid profit loss if the data subsidy level is low. For example, if the unit spectrum cost is $c=0.4$, the spectrum price discount rate is $\delta=0.2$ and the data subsidy amount is $\hat{d}=0.15\bar{d}$, then UW increases more than twice over without MP loss.

Even though the data subsidy scheme can bring UW increase without MP loss, the regulator should also check another important factor; UW increase has to be as high as the regulator's expenses (i.e., RR decrease) on that increase. For this, we plot Figure 8. The figure shows that the amount of UW increase is larger than the RR decrease for all data subsidy levels.[2] Therefore, we can conclude the data subsidy scheme is a cost-effective way to increase UW.

We plot MP, UW and RR gains with varying unit spectrum cost c in Figure 9. Without loss of generality, we assume $\bar{p}=1$, $\alpha=1$, $\delta=0.2$ and $\hat{d}/\bar{d}=0.1$. The figure shows that all the gains become larger as c increases. At the higher spectrum cost, MNOs purchase a lower spectrum amount and raise service prices. Then, some users cannot afford to consume the network services, which leads to MP and UW decreases. By giving the spectrum discount and the data subsidy, however, MNOs increase the purchased spectrum amount and more users consume the network services for free or at a reduced price. Also, the spectrum discount positively effects RR gain by the sale of a large spectrum amount at a low unit margin of revenue. Therefore, all the gains become larger as c increases. Interestingly, the spectrum cost has been increasing recently, so the data subsidy scheme is timely and needs to be considered as a measure.

[2]By plotting the ratio of UW increase to RR decrease under various conditions, we have verified that the amount of UW increase is larger than the RR decrease under almost all conditions, excepting when the unit spectrum cost c is very low.

VI. Extension to Oligopoly

In this section, we extend the results from the duopoly competition into the multiple MNO case (i.e., oligopoly competition). Let N denote the number of MNOs. Then, the decision maker index is $i \in \{1,2,\cdots,N\}$. On the other hand, its competitors index $-i$ becomes a set, which includes all the MNOs except MNO i. For the convenience of mathematical expressions, we define another set $-i^l$, which includes the competitors whose service price levels are lower than MNO i's. In the oligopoly competition, the user demand for MNO i changes from Equation (2):

$$d_i\left(p_i,p_{-i}\right)=\begin{cases}\bar{d}\left(1-\frac{p_i}{\bar{p}}\right),\text{if} p_i<p_j \text{ for} j\in -i,\\ \frac{w_i}{w_i+\sum\limits_{j\in -i}w_j}\bar{d}\left(1-\frac{p_i}{\bar{p}}\right),\text{if} p_i=p_j \text{ for} j\in -i,\\ \bar{d}\left(1-\frac{p_i}{\bar{p}}\right)-\alpha\sum\limits_{j\in -i^l}w_j,\ \text{otherwise.}\end{cases} \tag{13}$$

Following several steps similar to the proofs of Propositions 1, 2 and 3, we can find the SPE in the oligopoly competition as follows:

Proposition 5: *In the two-stage oligopoly competition, there is a subgame perfect equilibrium (w_O^{SPE},p_O^{SPE}):*

$$\begin{aligned}w_O^{SPE}&=\left(w_{O;1}^{SPE},\cdots,w_{O;N}^{SPE}\right)\\&=\left(\frac{\bar{d}}{(N+1)\alpha}\left(1-\frac{c}{\alpha\bar{p}}\right),\cdots,\frac{\bar{d}}{(N+1)\alpha}\left(1-\frac{c}{\alpha\bar{p}}\right)\right),\end{aligned}$$

$$\begin{aligned}p_O^{SPE}&=\left(p_{O;1}^{SPE},\cdots,p_{O;N}^{SPE}\right)\\&=\left(\frac{\bar{p}}{N+1}\left(1+\frac{Nc}{\alpha\bar{p}}\right),\cdots,\frac{\bar{p}}{N+1}\left(1+\frac{Nc}{\alpha\bar{p}}\right)\right).\end{aligned}$$

Proof: First, we check whether the SPE represents a NE in the Bertrand stage:

$$\begin{aligned} p_{O;i}^{SPE} &= \frac{\bar{p}}{N+1}\left(1+\frac{Nc}{\alpha\bar{p}}\right) \\ &= \bar{p}\left(1-\frac{N}{N+1}+\frac{Nc}{(N+1)\,\alpha\bar{p}}\right) \\ &= \bar{p}\left(1-\frac{\alpha}{\bar{d}}\left(\frac{N\bar{d}}{(N+1)\,\alpha}\left(1-\frac{c}{\alpha\bar{p}}\right)\right)\right) \\ &= \bar{p}\left(1-\frac{\alpha\left(w_{O;i}^{SPE}+\sum\limits_{j\in -i} w_{O;j}^{SPE}\right)}{\bar{d}}\right). \end{aligned} \tag{14}$$

Using Equations (13) and (14), we get the following equation.

$$\begin{aligned} d_i\left(p_{O;i}^{SPE}, p_{O;-i}^{SPE}\right) &= \frac{w_{O;i}^{SPE}}{w_{O;i}^{SPE}+\sum\limits_{j\in -i} w_{O;j}^{SPE}}\bar{d}\left(1-\frac{p_{O;i}^{SPE}}{\bar{p}}\right) \\ &= \alpha w_{O;i}^{SPE}. \end{aligned} \tag{15}$$

Equation (15) shows the demand for MNO i is equal to its network capacity (market clearing). In this case, even if MNO i lowers or raises its price, its revenue will not increase for the same reason as the duopoly case (Proposition 1). Therefore, the SPE represents a NE in the Bertrand stage.

Second, we check whether the SPE also represents a NE in the Cournot stage. Similar to the duopoly case, the profit maximization problem in the oligopoly Cournot competition is

$$\max_{w_{O;i}} \quad \alpha w_{O;i}\bar{p}\left(1-\frac{\alpha\left(w_{O;i}+\sum\limits_{j\in -i} w_{O;j}\right)}{\bar{d}}\right)-cw_{O;i}$$

The only difference from the duopoly case is that the competitor index $-i$ becomes a set. The objective function is a quadratic function of which the optimal solution ($w_{O;i}^{*}$) is

$$w_{O;i}^{*}=\frac{\bar{d}}{2\alpha}\left(1-\frac{c}{\alpha\bar{p}}\right)-\frac{1}{2}\left(\sum_{j\in -i} w_{O;j}\right). \tag{16}$$

By the fact that a NE is an intersection of all players' optimal strategies, we can find the condition for becoming a NE ($w_{O;i}^{NE}$) in the Cournot stage as follows:

$$w_{O;i}^{NE}=\frac{\bar{d}}{(N+1)\,\alpha}\left(1-\frac{c}{\alpha\bar{p}}\right). \tag{17}$$

The SPE satisfies this condition. Therefore, the SPE also represents a NE in the Cournot stage. ■

With Proposition 5, we calculate MP, UW and RR as follows:

$$\mathrm{MP}_O=\frac{N}{(N+1)^2}\bar{p}\bar{d}\left(1-\frac{c}{\alpha\bar{p}}\right)^2, \tag{18}$$

$$\mathrm{UW}_O=\frac{1}{2}\left(\frac{N}{N+1}\right)^2\bar{p}\bar{d}\left(1-\frac{c}{\alpha\bar{p}}\right)^2, \tag{19}$$

$$\mathrm{RR}_O=\frac{N}{N+1}\frac{c\bar{d}}{\alpha}\left(1-\frac{c}{\alpha\bar{p}}\right). \tag{20}$$

where subscript O denotes the oligopoly competition. From above equations, we observe that UW and RR increase with the number of MNOs in the oligopoly. On the other hand, MP decreases. This is because the competition among the MNOs drives each MNO to purchase more spectrum and to lower its service price to attract more users (Proposition 5). Therefore, the regulator can improve UW by facilitating the entry of new MNOs even though it leads to a decrease in the MNOs' total profits.

In the oligopoly competition, we apply the data subsidy scheme for improving UW. We can find a SPE in the data subsidy scheme as follows:

Proposition 6: *In the two-stage oligopoly competition, if the regulator adopts the data subsidy scheme, then there is a subgame perfect equilibrium (w_{OD}^{SPE},p_{OD}^{SPE}):*

$$\begin{aligned} w_{OD}^{SPE} &= \left(w_{OD;1}^{SPE},\cdots,w_{OD;N}^{SPE}\right) \\ &= \left(\tfrac{\bar{d}-\hat{d}}{(N+1)\alpha}\left(1-\tfrac{(1-\delta)c}{\alpha\bar{p}}\right),\cdots,\tfrac{\bar{d}-\hat{d}}{(N+1)\alpha}\left(1-\tfrac{(1-\delta)c}{\alpha\bar{p}}\right)\right), \end{aligned}$$

$$\begin{aligned} p_{OD}^{SPE} &= \left(p_{OD;1}^{SPE},\cdots,p_{OD;N}^{SPE}\right) \\ &= \left(\tfrac{\bar{p}}{N+1}\left(1+\tfrac{N(1-\delta)c}{\alpha\bar{p}}\right),\cdots,\tfrac{\bar{p}}{N+1}\left(1+\tfrac{N(1-\delta)c}{\alpha\bar{p}}\right)\right). \end{aligned}$$

Proof: In the data subsidy scheme, the maximum demand for charged network services and the unit spectrum cost become $\bar{d}-\hat{d}$ and $(1-\delta)c$, respectively. Excepting this change, steps for finding a SPE in the data subsidy scheme are equal to that in the conventional scheme (Proposition 5). Therefore, we can find a SPE in the data subsidy scheme by replacing $\bar{d}$ with $\bar{d}-\hat{d}$ and c with $(1-\delta)c$ in Proposition 5. ■

With Proposition 6, we calculate MP, UW and RR in the data subsidy scheme as follows:

$$\mathrm{MP}_{OD}=\frac{N}{(N+1)^2}\bar{p}(\bar{d}-\hat{d})\left(1-\frac{(1-\delta)c}{\alpha\bar{p}}\right)^2, \tag{21}$$

$$\mathrm{UW}_{OD}=\frac{1}{2}\left(\frac{N}{N+1}\right)^2\bar{p}(\bar{d}-\hat{d})\left(1-\frac{(1-\delta)c}{\alpha\bar{p}}\right)^2+\frac{1}{2}\bar{p}\hat{d}, \tag{22}$$

$$\mathrm{RR}_{OD}=\frac{N}{N+1}\frac{(1-\delta)c(\bar{d}-\hat{d})}{\alpha}\left(1-\frac{(1-\delta)c}{\alpha\bar{p}}\right). \tag{23}$$

where subscript OD denotes the data subsidy scheme in the oligopoly competition.

To show the competition and the data subsidy effects jointly, we plot Figure 10. Without loss of generality, we assume $\bar{p}=1$, $\alpha=1$, $\delta=0.2$ and $\hat{d}/\bar{d}=0.1$. As noted, the

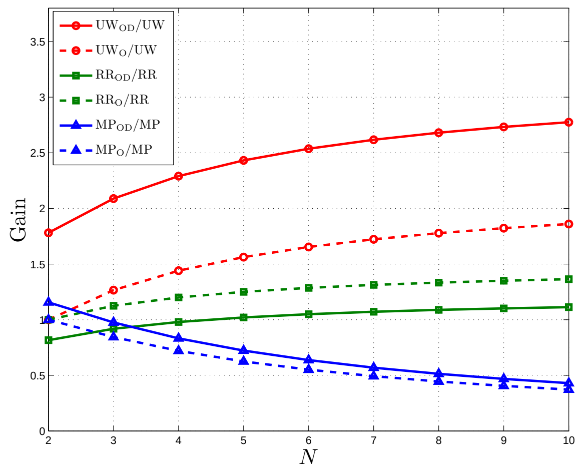

Fig. 10. MNO profit ($\mathrm{MP_O/MP}$ and $\mathrm{MP_{OD}/MP}$), user welfare ($\mathrm{UW_O/UW}$ and $\mathrm{UW_O/UW_{OD}}$) and regulator revenue ($\mathrm{RR_O/RR}$ and $\mathrm{RR_{OD}/RR}$) gains as a function of the number of MNOs N. Without loss of generality, we assume $\bar{p} = 1$, $\alpha = 1$, $\delta = 0.2$ and $\hat{d}/\bar{d} = 0.1$.

figure shows that more competition increases UW and RR while it decreases MP. An interesting observation is that the UW gain from competition is relatively low. For example, even if the regulator allows the entry of 8 additional MNOs (i.e., $N = 10$) in the duopoly competition, he cannot achieve 2-fold UW gain. On the other hand, if he adopts the data subsidy scheme, then he can accomplish it by just adding one more MNO (i.e., $N = 3$). In that case, MP and RR decreases are marginal as in Figure 10. This means the regulator can improve UW more efficiently by allowing the entry of new MNOs and adopting the data subsidy scheme jointly.

VII. Conclusions

In this paper, we show that giving a price subsidy to users may not necessarily lead to an increase of user welfare in mobile communication services. Therefore, we suggest a data subsidy scheme as an alternative, where the regulator offers a spectrum price discount to MNOs in return for providing a predefined data amount to users free of charge. To mathematically analyze the subsidy effect, we formulate and solve optimization problems by applying the two-stage Cournot and Bertrand competition model that is well understood in microeconomics. An interesting result is that the data subsidy scheme can increase user welfare even further without MNO profit loss. Moreover, the increasing amount of user welfare is higher than the regulator's expenses for implementing the data subsidy scheme.

In recent years, governments around the world released their plans to make more spectrum available for mobile broadband use. Therefore, it is very important to create an efficient policy regarding new spectrum allocation that will improve user welfare. The data subsidy scheme deserves thoughtful consideration. Even though our analytic results are derived under some assumptions for mathematical tractability, it will provide useful insight for development of an efficient spectrum allocation policy regarding mobile communication services.

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